

## Computer modeling of machine parts

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The article describes a computational modeling machine and its parts. For example, the appropriate procedure, numerical solutions and the comparison with the calculation carried out analytically are described.

The general procedure is to create an appropriate model, determining the number of degrees of freedom, the mathematical description (at the discretion of the weight and geometric quantities, types of movements, types of strain, the type of drive and load, respectively other parameters).

The resulting equations are then solved numerically by the program Matlab. As an example, it is stated in the article the numerical solution of plane stress distribution of rotating wheel. The result is the calculation of a radial and tangential stress  $\sigma_r$ ,  $\sigma_t$  for each disc form. The simplest shape is the wheel disc of constant thickness (example 1).

Another form of N wheel divides the cylindrical rings and is determined by calculating the voltage on the outer ring, which is the default stress of the next ring (example 2). The replacement profile side wall ring hyperboloid refines the previous calculation and the normal stress and tangential stress  $\sigma_r$ ,  $\sigma_t$  is along the increasing radius of the smallest disk (example 3).

The constantly increasing speed of the rotating parts of the machinery, increasing normal stress and tangential stress  $\sigma_r$ ,  $\sigma_t$ . This article aims to show the possibility of a suitable design model disc form and numerical solution of the normal stress and tangential stress are at a minimum.

### 1. Introduction

Of the known boundary values of the hyperbolic k-th ring (Fig. 1) is calculated for the change of its thickness h are

$$h_k = a \cdot r_k^{-b}, h_{k+1} = a \cdot r_{k+1}^{-b} \quad (1)$$

where exponent of the hyperbola b is

$$b = \frac{\ln(h_k) - \ln(h_{k+1})}{\ln(r_{k+1}) - \ln(r_k)} \quad (2)$$

where value for the exponents  $b$  are for the ring: constant thickness  $b=0$ , widening  $b>0$  and narrowing  $b<0$ .

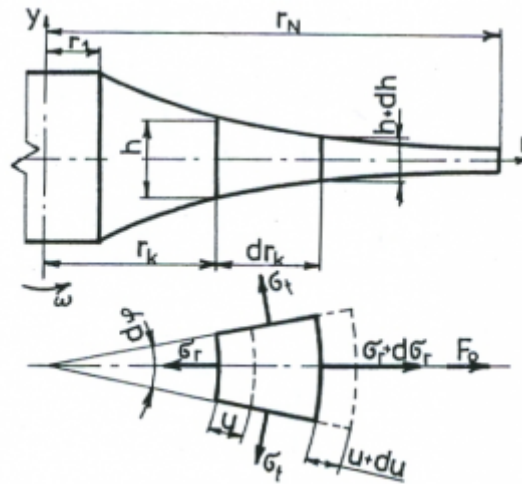


Figure 1: The out of joint element profile wheels

## 2. Calculation of stress $\sigma$ and deformation $u$

Under the terms of the balance of forces in  $r$  direction (Fig. 1) we obtain

$$(\sigma_r + d\sigma_r)(r + dr)(h + dh)d\varphi - \sigma_r r d\varphi h - \sigma_t r d\varphi h + \rho dr d\varphi h \omega^2 r = 0 \quad (3)$$

Editing (3) we obtain

$$\frac{d\sigma_r}{dr} + \sigma_r \frac{d(\ln(h))}{dr} - \frac{\sigma_t - \sigma_r}{r} = -\rho \omega^2 r \quad (4)$$

where  $\rho$  is density and  $\omega$  is angular velocity. The strains are defined

$$\begin{aligned} \varepsilon_r &= \frac{du}{dr} \\ \varepsilon_t &= \frac{u}{r} \end{aligned} \quad (5)$$

where  $u$  is deformation.  $\mu$  is Poisson's ratio. and generalized Hooke's law [1] is

$$\begin{aligned} \varepsilon_r &= \frac{1}{E}(\sigma_r - \mu\sigma_t) \\ \varepsilon_t &= \frac{1}{E}(\sigma_t - \mu\sigma_r) \end{aligned} \quad (6)$$

Where  $\mu$  is Poisson's ratio. The equations (4), (5) can be calculated

$$\begin{aligned} \sigma_r &= \frac{1}{1-\mu^2} \left( \frac{du}{dr} + \mu \frac{u}{r} \right) \\ \sigma_t &= \frac{1}{1-\mu^2} \left( \frac{u}{r} + \mu \frac{du}{dr} \right) \end{aligned} \quad (7)$$

Substituting (7) into equation (4) we obtain for  $k$ -th ring true

$$\frac{d^2 u}{dr_k^2} + \left( \frac{1}{r_k} + \frac{d(\ln(h))}{dr_k} \right) \frac{du}{dr_k} - \left( 1 - \mu r_k \frac{d(\ln(h))}{dr_k} \right) \frac{u}{r_k^2} = -Ar_k$$

$$A = \rho\omega^2 \frac{1-\mu^2}{E} \quad (8)$$

where E is Young's modulus. Substituting (7) into equation (4) we obtain

$$\frac{d^2u}{dr_k^2} + \frac{1}{r_k}(1-b)\frac{du}{dr_k} - (1-\mu b)\frac{u}{r_k^2} = Ar_k \quad (10)$$

solution of the equation (10) is deformation

$$u = C_1 r_k^{\lambda_1} + C_2 r_k^{\lambda_2} - \frac{A}{8-b(3+\mu)} r_k^3$$

$$\lambda_{1,2} = -\frac{1}{2} \pm \sqrt{\left[\frac{b^2}{4} + (1+\mu b)\right]} \quad (11)$$

Substituting (11) into (7) we obtain

$$\sigma_r = \frac{E}{1-\mu^2} \left[ C_1 r_k^{\lambda_1-1} (\lambda_1 + \mu) + C_2 r_k^{\lambda_2-1} (\lambda_2 + \mu) - \frac{A}{8-b(3+\mu)} r^2 (3 + \mu) \right]$$

$$\sigma_t = \frac{E}{1-\mu^2} \left[ C_1 r_k^{\lambda_1-1} (1 + \lambda_1 \mu) + C_2 r_k^{\lambda_2-1} (1 + \lambda_2 \mu) - \frac{A}{8-b(3+\mu)} r^2 (1 + 3\mu) \right] \quad (12)$$

We calculated the constants  $C_1, C_2$  from the boundary conditions

$$\sigma_{r(r_1)} = p_1, \sigma_{r(r_N)} = p_N \quad (13)$$

where  $p_1, p_N$  are known pressures on the wheel radius  $r_1, r_N$  (Fig. 1)

### 3. Results

With Matlab [1], [2], [3] it is possible to calculate curves  $\sigma_r, \sigma_t$  for examples 1, 2, 3 (Fig.2). Parameters are the wheels:

$$\sigma_{r(r_1)} = 7.5MPa, \sigma_{r(r_N)} = 20MPa, r_1 = 0.1m, r_N = 0.5m, \omega = 314s^{-1}, \rho = 7800kgm^{-3}, \mu = 0.3, E = 2.10^5MPa$$

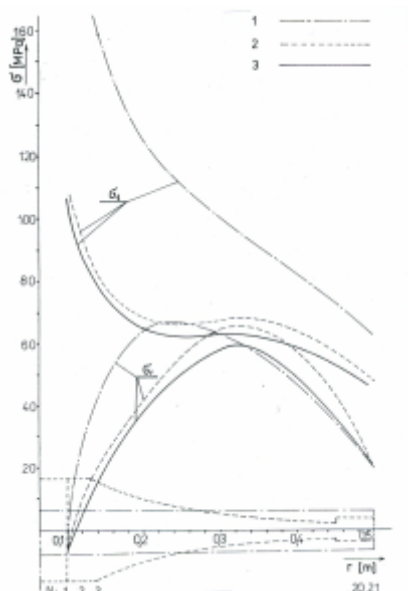


Figure 2: The stress  $\sigma_r$ ,  $\sigma_t$  for examples 1, 2, 3

The curves in Fig. 2 show that the tension is the smallest at the wheels with the hyperbolic profile.

## References

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