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Cooling of an electric conductor by free convection – analytical, computational and experimental approaches

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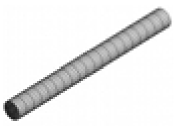

This article deals with cooling analyses of horizontally arranged bare Cu electric conductor using analytical and numerical approaches. Results of these analyses will be compared to the ones obtained from experimental measurement.

1. Problem description

From time immemorial, the maximum continuous operating performance and current loading capacity of cables and wires is important criterion in the design of cable systems (materials, geometry, construction) and design of power electrical installation (e.g. deposit method). The passing electrical current by means of Joule losses heats up the conductor and therefore its temperature and temperature of its electrical insulation system exceeds the ambient temperature. This temperature depends quadratically on the passing current. Long-term exceeding of projected operating temperatures of cable conductors causes significantly faster aging of insulation, increased corrosion of cable cores in extreme cases and deterioration of mechanical properties of insulation.

In recent years there has been rapid development of computational methods, models and simulations, which allows, as will be shown, to determine the steady state temperature of the current passing bare conductor. The proposed elementary model consists of horizontally arranged bare electric copper conductor (thermal conductivity 401 $Wm^{-1}K^{-1}$) with diameter of 1.48 mm. The conductor carried AC current according to measurement order from 5 to 30 A RMS. Cooling of the conductor was only due to free convection and radiation effects (ambient temperature was 22 °C). Experimental measurements were performed to determine reference surface temperatures of the conductor under steady thermal-electric state. These values were compared to the temperatures obtained from analytical calculation and numerical simulation.

2. Approaches for determining of cooling of electric conductor

In next chapters there will be shown three principled approaches to determining of cooling of electric conductor via free convection (analytical and numerical solution and experimental measurement).

2.1 Analytical solution

This solution results from Fourier differential equation of thermal conductivity that was extended by radiation and convection effect [1]. Convection coefficient was calculated according to criterion equations and it was set up as temperature dependent variable. Boundary conditions were Joule heat generated in conductor by electric losses, radiation and convection conditions. Finally, surface temperature of the conductor was obtained. Physical quantities:

- c – specific heat of conductor [$Jkg^{-1}K^{-1}$]
- $c_{p\,air}$ – air specific heat (constant pressure) [$Jkg^{-1}K^{-1}$]
- d – diameter of conductor [m]
- g – gravity [ms^{-2}]
- Gr – Grashoff number [-]
- I – electric current [A]
- L – characteristic dimension of conductor [m]
- Nu0 – Nusselt number (starting value) [-]
- Nu – Nusselt number [-]
- Pr – Prandtl number [-]
- r – radius of conductor [m]
- R – conductor resistivity [Ω]
- Ra – Rayleigh number [-]
- S – cross-section area of conductor [m^2]
- t – surface temperature of conductor [°C]
- T – surface temperature of conductor [K]
- t_{amb} – ambient temperature [°C]
- T_{amb} – ambient temperature [K]
- ΔT – temperature difference between conductor surface and ambient [K]
- V – volume of conductor [m^3]
- α – convective heat transfer coefficient [$Wm^{-2}K^{-1}$]
- β_{air} – air expansion coefficient [K-1]
- ε – emissivity [-]
- λ – thermal conductivity of conductor material [$Wm_{-1}K_{-1}$]
- λ_{air} – air thermal conductivity [$Wm^{-1}K^{-1}$]
- μ_{air} – air dynamic viscosity [Nsm^{-2}]
- ν_{air} – air kinematic viscosity [m^2s^{-1}]
- ρ – density of conductor material [kgm^{-3}]
- ρ_e – electric resistivity [Ωm]
- ρ_{air} – air density [kgm_{-3}]
- σ – Stephan-Boltzman constant [5.6704×10^{-8} $Wm^{-2}K^{-4}$]
- τ – time [s]
- ϕ – heat flux from the surface of conductor [Wm^{-2}]
- ϕ_K – heat flux for convection [Wm^{-2}]
- ϕ_R – heat flux for radiation [Wm^{-2}]
- Φ_V – heat generated in conductor [Wm^{-3}]

General equation for heat transfer in solid materials is called Fourier-Kirchhoff law

$$\frac{\partial t}{\partial \tau} = \frac{\lambda}{c\rho}\Delta^2 t + \frac{\Phi_v}{c\rho} \tag{1}$$

For steady state temperature doesn't depend on time

$$\frac{\partial t}{\partial \tau} = 0 \tag{2}$$

Applying (2) into the (1) we obtain Poisson equation

$$\Delta^2 t = \frac{\Phi_v}{\lambda} \tag{3}$$

Our conductor that carries the electric current is horizontally arranged cylinder, so it is necessary to transform (3) into the cylindrical coordinate system

$$\frac{d^2 t}{dr^2} + \frac{1}{r}\frac{dt}{dr} = -\frac{\Phi_v}{\lambda} \tag{4}$$

Solution of this second-order differential equation has form,

$$t = -\frac{\Phi_v}{\lambda}\frac{r^2}{4} + c_1 ln(r) + c_2 \tag{5}$$

where temperature is primary unknown. Cross-section area of conductor is solid circle therefore r goes from 0 to final radius so term containing ln(r) has to be eliminated. We determine $c_1 = 0$. Then (5) changes into (6).

$$t = -\frac{\Phi_v}{\lambda}\frac{r^2}{4} + c_2 \tag{6}$$

Integration constant c^2 is evaluated according to known heat flow from surface of conductor. This heat flow is defined by sum of convective and radiation heat flows from conductor surface. For convection it is,

$$\varphi_K = \alpha(t - t_{ok}) \tag{7}$$

for radiation it is

$$\varphi_R = \varepsilon\sigma(T^4 - T^4_{ok}) \tag{8}$$

Heat transfer from conductor surface is described by Fourier law

$$\varphi = -\lambda\left(\frac{dt}{dr}\right)_{r=\frac{d}{2}} \tag{9}$$

Boundary condition for surface of conductor is

$$\varphi = \varphi_K + \varphi_R \tag{10}$$

After some math operations we get iterative rule for calculation surface temperature (indexes [i], [i+1] represent iterative steps)

$$t_{[i+1]} = \frac{\Phi_v}{\lambda}\frac{d}{4} - \frac{\varepsilon\sigma(T^4_{[i]} - T^4_{ok})}{\alpha} + t_{ok} \tag{11}$$

Heat generated in conductor is calculated according to Joule heat looses, so it is necessary to calculate resistance of conductor

$$R = \rho_e \frac{L}{S} \tag{12}$$

Resistivity is temperature depended variable. For cooper conductor (ρ_e 20°C = 1.69×10^{-8} Ωm) we obtained temperature dependency in this form

$$\rho_e = 7.2875.10^{-11}t + 1.5483.10^{-8} \tag{13}$$

Then heat generated in conductor is

$$\Phi_v = \frac{RI^2}{V} = \frac{16\rho_e I^2}{\pi^2 d^4} \tag{14}$$

Calculation of convective heat transfer coefficient is more complicated. This calculation is based on empirical equations those will be discussed in next section. Convective heat transfer coefficient is calculated according to empirical equation based on Nusselt number and it is temperature dependent variable

$$\alpha = \frac{Nu\lambda_{vz}}{L} = f(t) \tag{15}$$

Characteristic dimension for horizontal cylinder is [2]

$$L = \pi \frac{D}{2} \tag{16}$$

Nusselt number is the ratio of convective to conductive heat transfer across the boundary and it is calculated using next equations [2]

$$\begin{gathered} Nu_0 = 0.36\pi \\ Nu = \left[Nu_0^{\frac{1}{2}} + Ra^{\frac{1}{6}} \left(\frac{f(Pr)}{300} \right)^{\frac{1}{6}} \right]^2 \\ f(Pr) = \left[1 + \left(\frac{0.5}{Pr} \right)^{\frac{9}{16}} \right]^{\frac{-16}{9}} \end{gathered} \tag{17}$$

The Rayleigh number is defined as the product of the Grashof number, which describes the relationship between buoyancy and viscosity within a fluid, and the Prandtl number, which describes the relationship between momentum diffusivity and thermal diffusivity. Hence the Rayleigh number itself may also be viewed as the ratio of buoyancy and viscosity forces times the ratio of momentum and thermal diffusivities. For free convection around horizontally arranged cylindrical conductor these equations are

$$\begin{gathered} Ra = GrPr \\ Gr = \frac{g\beta_{vz}\Delta TL^3}{\nu_{vz}^2} \\ Pr = \frac{c_{pvz}\mu_{vz}}{\lambda_v z} \end{gathered} \tag{18}$$

Air kinematic viscosity is based on dynamic viscosity and density of air and all these material properties including air thermal conductivity are temperature dependent variables

$$\nu_{vz} = \frac{\mu_{vz}}{\rho_{vz}} \tag{19}$$

Air properties for t = 20 °C are presented in Tab. 1.

Tab. 1. Air properties for t = 20 °C

$c_{p\,air}$ [$Jkg^{-1}K^{-1}$]	Pr [-]	β_{air} [K^{-1}]	μ_{air} [Nsm^{-2}]	ν_{air} [m^2s^{-1}]	λ_{air} [$Wm^{-1}K^{-1}$]	ρ_{air} [kgm^{-3}]
1.007×10^{3}	0.7083	3.43×10^{-3}	1.811×10^{-5}	1.527×10^{-5}	2.589×10^{-2}	1.186

Temperature dependencies for these air properties were calculated for specified temperature range 22-127 °C, see Fig. 1. According to (15) convective heat transfer coefficient was calculated as temperature dependent variable. Approximation function for α_{approx} was obtained as (see Fig. 1)

$$\alpha_{approx} = 8.477 ln(t) - 2.166 \tag{20}$$

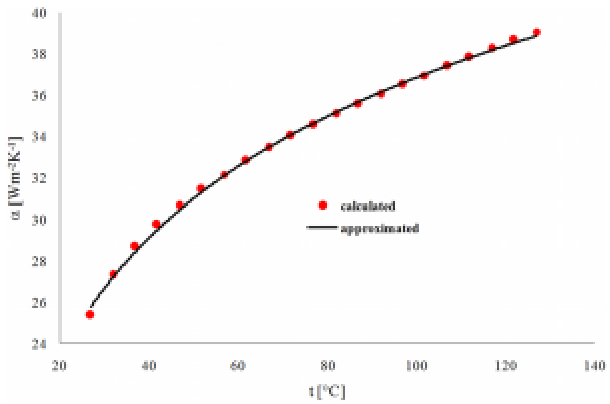

Fig. 1. Calculated and approximated values of α

Next unknown parameter in (11) that describes conductor surface temperature is coefficient of emissivity ε. For our case the value of emissivity was chosen in accordance to [3] for cooper polished surface ε = 0.07. Now it is possible to calculate iterative rule (11) for conductor surface temperature for chosen range of electric currents, see Tab. 2.

Tab. 2. Conductor surface temperature – analytical solution

I [A]	$T_{analytical}$ [°C]
5	24.14
10	30.15
15	39.54
20	52.29
25	68.80
30	89.84

2.2 Numerical solution

To obtain numerical solution ANSYS Workbench program was used, where steady-state electro-thermal (abbr. E-T) simulation was performed. The model and all boundary conditions were created according to the analytical solution. Material properties of conductor and convective heat transfer coefficient were set as temperature dependent variables. Mesh of finite electro-thermal 3D elements was created in the software, see Fig. 2. Surface temperature of the conductor was obtained directly from the software, see Fig. 2. Results from this numerical solution are in Tab. 3.

Tab. 3. Conductor surface temperature – numerical solution

I [A]	T_{E-T} [°C]
5	24.10
10	30.12
15	39.51
20	52.32
25	68.85
30	89.80

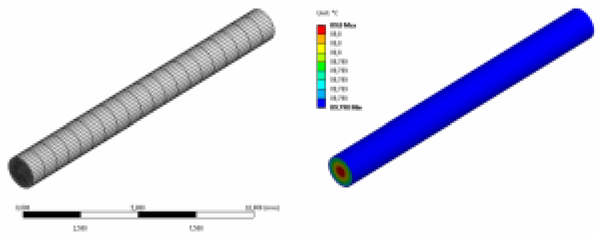

Fig. 2. Mesh of the conductor (left), temperature of the conductor (right)

2.3 Experimental setup

The experimental results proposed in this paper were achieved by loading characteristic measurements for horizontal Cu bare conductor, diameter 1.48 mm. Current in the measurement circuit (5, 10, 15, 20, 25 and 30 Amps AC) was regulated by autotransformer (AT) and current transformer (PT) (see Fig. 3 for connection details) and measured by analog ammeter (A) (precision 0.5 %). Conductor surface temperature was measured by K-type thermocouple and real time logged into computer by Fluke 289 multimeter (MT) and support logging software Fluke View Forms (PC), see Fig. 4. Time logging interval was set to 1 second. Temperature measurement precision was 1%. Ambient temperature during measurement was 22 °C. Results from measurement are in Tab. 4.

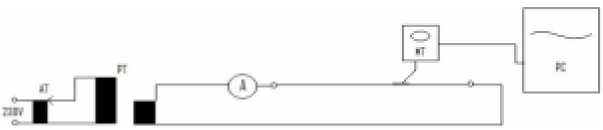

Fig. 3. Schematic illustration of the experimental setup.

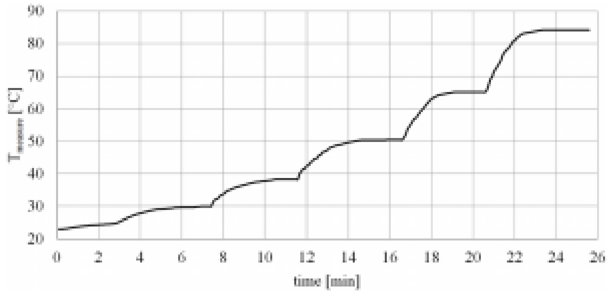

Fig. 4. Time evolution of conductor surface temperature (loading currents: 5, 10, 15, 20, 25, 30 A)

Tab. 4. Conductor surface temperature – measurement

I [A]	**T_{E-T} [°C]**
5	24.60
10	30.00
15	38.30
20	50.40
25	65.62
30	84.24

3. Comparison and conclusion

In Tab. 5 there are differences of temperature for the individual solutions in comparison to measurement. Fig. 5 shows calculated and measured surface temperatures in graphic form.

Tab. 5. Differences of conductor surface temperatures for the individual approaches

I [A]	**$T_{measure}$ (°C)**	**$\Delta T_{analytical}$ (%)**	**ΔT_{E-T} (%)**
5	24.60	-1.89	-2.03
10	30.00	0.49	0.40
15	38.30	3.23	3.15
20	50.40	3.75	3.81
25	65.62	4.85	4.92
30	84.24	6.65	6.60

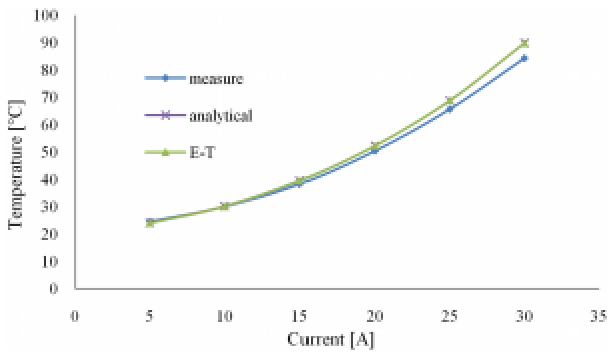

Fig. 5. Graph of surface temperature of the wire for considered models

Results from individual solutions were compared to measured temperature data. Deviations between calculations and measurement were in acceptable range. Analytical solution was relatively accurate and simple but unusable for models with complex geometry. More appropriate way to calculate surface temperature of the conductor is using the ANSYS Workbench environment. There is possibility to create complex geometry (if the convective heat transfer coefficient is possible to define adequately) and data- and time- intensiveness is not so high.

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