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PI controller design method with desired phase margin and settling time

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In this paper controller design method for SISO systems with performance specification in term of phase margin and desired settling time is presented. Mentioned method is based on modification of Neimark D-partition method which ensure desired phase margin of open loop and desired settling time by setpoint change. The developed frequency domain design technique is graphical, interactive and insightful. Theoretical results are demonstrated on examples.

1. Introduction

Frequency domain techniques for analysis and controller design dominate SISO control system theory. Bode, Nyquist, Nichols, and root locus are the usual tools for SISO system analysis.
Frequency methods are often used for controller design because it is easy to ensure performance of closed loop system through phase margin in open loop (Nagurka and Yaniv, 2003). To achieve the desired phase margin, controllers are usually designed using Bode characteristics (Fung, et al 1998),(Ho, et al 1995).

In this paper Neimarks method of D-partition is used (Neimark, 1992). This method is usually used for controller design which ensures closed loop stability with desired stability degree. In this paper is presented modification of this method aimed on phase margin instead of stability degree. Settling time is ensured using crossover frequency which has significant influence on this second performance specification.

2. Preliminaries and problem formulation

Consider classical feedback system depicted in Fig. 1.

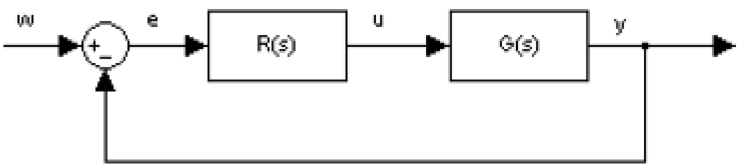

Figure 1: Classical feedback system

The aim is to design PI controller for plant using Neimark D-partition method so, that not only stability will be ensured but performance in term of phase margin and desired settling time too.

3. Theoretical results

Consider closed loop feedback system with PI controller and SISO plant.

$$R(s) = k + \frac{k_i}{s} \qquad G(s) = \frac{B(s)}{A(s)} \tag{1}$$

Substituting (1) into characteristic function (2)

$$1 + R(s)G(s) = 0 \tag{2}$$

and after small manipulations is possible to obtain:

$$k + \frac{k_i}{s} = -1\frac{A(s)}{B(s)} \tag{3}$$

After other substitution $s = j\omega$ and division into real and imaginary part:

$$\begin{aligned} k &= real\left\{(-1+0j)\frac{A(j\omega)}{B(j\omega)}\right\} \\ k_i &= imag\left\{(-1+0j)\frac{A(j\omega)}{B(j\omega)}(-\omega)\right\} \end{aligned} \tag{4}$$

If ω is changing step by step in interval $\omega \in (0, \infty)$ from real part of (4) is possible to calculate frequency dependent vector of complex numbers which plotted in complex plane create D-curve for parameter k. Similar is it with imaginary part of (4) from which is possible obtain k_i. D-curve plot for parameters k and k_i (PI controller) can be done in one step. This way is possible to calculate and paint marginal D-curves. If PI controller parameters will be chosen from this D-curves system with controller will have Nyquist plot intersecting point -1+0j in complex plane (Figure. 2).

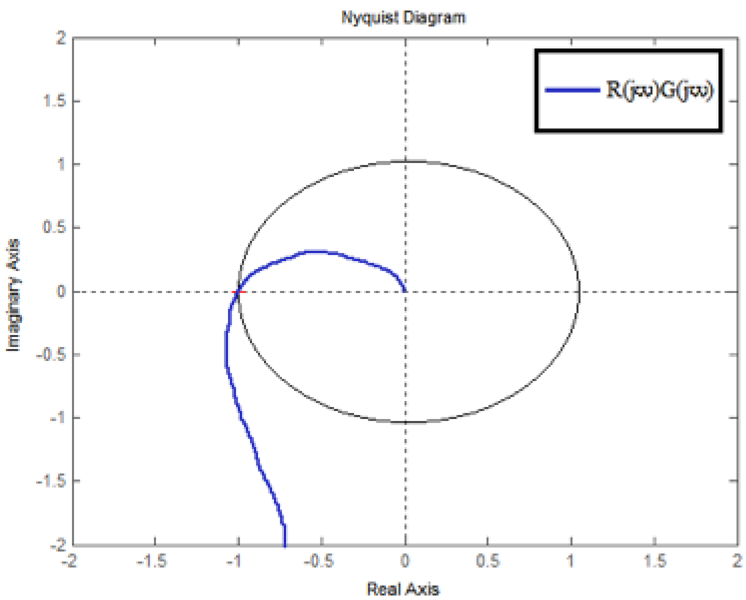

Figure. 2 Nyquist plot intersecting point -1+0j

To obtain any phase margin it is necessary to move intersection point with unit circle form point -1+0j to point situated in lower left quadrant on the same circle. To obtain desired phase margin φ, (2) have to be modified as follows:

$$(x+yj)+R(s)G(s)=0 \tag{5}$$

where values of x and y depends on φ (Figure 3)

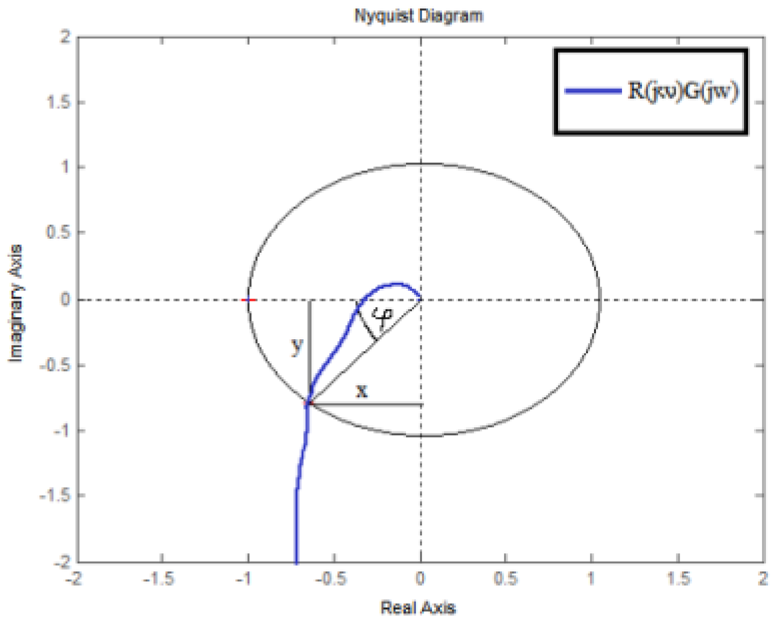

Figure. 3 Nyquist plot with intersection ensuring phase margin φ.

According to situation on Figure 3 and known radius of unit circle, values of x and y can be calculate as follows:

$$\begin{aligned} y &= sin\varphi \\ x &= \sqrt{1-y^2} \end{aligned} \tag{6}$$

For calculating D-curves of PI controller parameters k and k_i, ensuring desired phase margin φ (4) is modified to (7).

$$\begin{aligned} k &= real\left\{(-x+yj)\frac{A(j\omega)}{B(j\omega)}\right\} \\ k_i &= imag\left\{(-x+yj)\frac{A(j\omega)}{B(j\omega)}(-\omega)\right\} \end{aligned} \tag{7}$$

Settling time of controlled system depends on plant gain and values of controller parameters. Higher values of k_i controller parameter provide shorter settling time for the price of higher controller output. D-curves of plants with possitive gain, starts $(\omega=0)$ in negative values of both PI controller parameters not far from origin. If plant is possible to stabilize with chosen phase margin using PI control structure, with increasing frequency D-curves continues to area where both PI controller parameters has possitive values (Fig. 4).

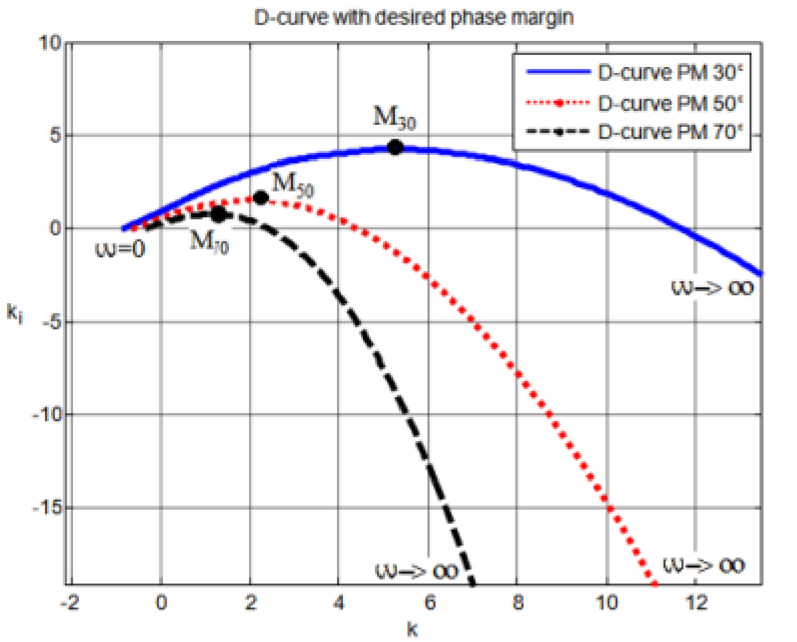

Fig. 4 PI controller D-curves for different phase margin

Chosing controller parameters from D-cruve also crossover frequency ω_c is chosen (frequency when open-loop system reaches unity circle). Controller parameters from D-curves maximum point M, yields shortest settling time. Consider parameters with higher frequency as maximum point frequency ω_M has no practical purpose. By increasing of crossover freqeuncy ω_c from ω_{min} (lowest frequency which yields possitive controller parameters) to ω_c, both controller parameters will increase and settling time T_s will decrease.

$$\omega_{c2} > \omega_{c1} \Rightarrow T_{s2} < T_{s1} \qquad \omega_{c1}, \omega_{c2} \in < \omega_{min}, \omega_M > \tag{8}$$

According to (8) it is possible to define controller design algorithm, ensuring desired phase margin and settling time consisting of following steps:

1. Chose desired phase margin φ and settling time T_s
2. Calculate x and y according to (6)
3. Calculate $k(\omega)$ and $k_i(\omega)$ frequency vectors from (7) and find ω_M as frequency where $k_i(\omega)$ has maximum value $max = k_i(\omega_M)$
4. Plot D-curve with k parameter on x-axis and k_i on y-axis and design temporary PI controller for any crossover frequency $\omega_{ctmp} \in (\omega_{min}, \omega_M)$
5. Measure settling time T_{s_tmp} of system with designed temporary controller
6. Calculate ω_c frequency ensuring desired settling time T_s as $\omega_c = \frac{T_{s_tmp}}{T_s}\omega_{c_tmp}$
7. If $\omega_c > \omega_M$ back to a) and chose higher T_s or lower φ, else design final PI controller for crossover frequency ω_c

4. Case study

Consider stable 3-th order plant

$$G(s) = \frac{2s+1}{s^3+3s^2+3s+1}$$

a) For this plant PI controller ensuring phase margin $\varphi = 50^o$ and settling time T_s=6s will be designed.

b) According to (6) and desired phase margin:

$$y = sin\varphi = 0.766$$
$$x = \sqrt{1-y^2} = 0.6428$$

c) D-curves for parameters k and k_i can be calculated in one step and plotted in one figure, because characteristic function is divided into real and imaginary part and each parameter is calculated from another part.

$$k = real\left\{(-0.6428 - 0.766j)\frac{A(j\omega)}{B(j\omega)}\right\}$$
$$k_i = imag\left\{(-0.6428 - 0.766j)\frac{A(j\omega)}{B(j\omega)}(-\omega)\right\} \quad (8)$$

d) In fig. 5 is plotted D-curve for parameters k and k_i calculated for φ. Parameter vector $k_i(\omega)$ reaches the maximum value 1.5569 by frequency $\omega_M = 1.73$ rad/s.

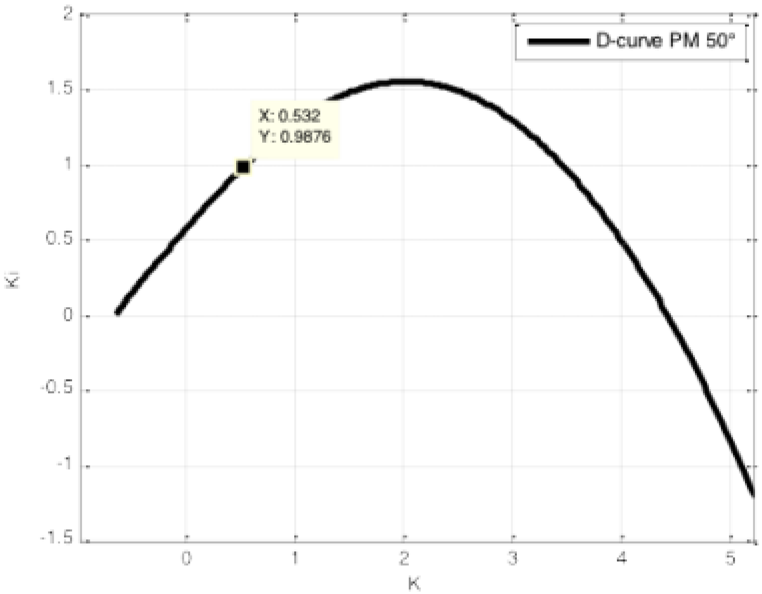

Fig. 5 D-curve for phase margin $\varphi = 50^o$

Temporary controller has following parameters:

$$R_{tmp}(s) = \frac{0.532s+0.9876}{s}$$

and from bode plot Fig. 6 it is possible to see that phase margin is really $\varphi = 50^o$ and crossover frequency $\omega_c = 0.92$ rad/s.

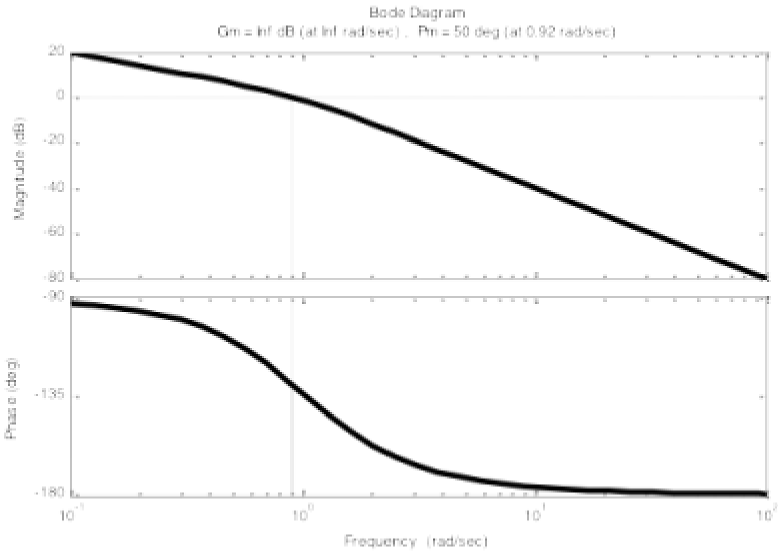

Fig. 6 Bode plot for open-loop system with temporary PI controller

e) Settling time is the time required for the output to reach and remain within a 2% error band following input stimulus. This way for system with temporary controller was measured settling time T_{s_tmp}=7.3s (Fig. 7).

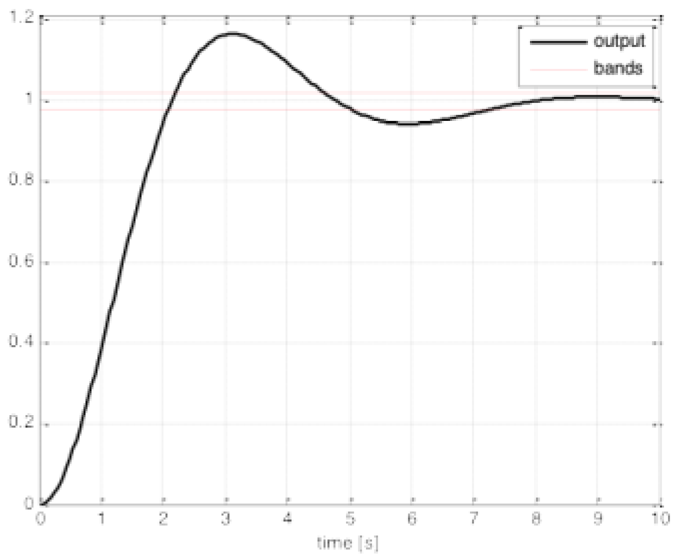

Fig. 7 Step response for closed loop system with temporary PI controller

f) Final crossover frequency ω_c is now calculated according to (9)

$$\omega_c = \frac{7.3}{6}0.92 = 1.193 \text{ rad/s}$$

g) Formula does not parse rad/s

From Fig. 8 is possible to see that phase margin is equal to desired phase margin φ and also crossover frequency is very close to calculated frequency ω_c.

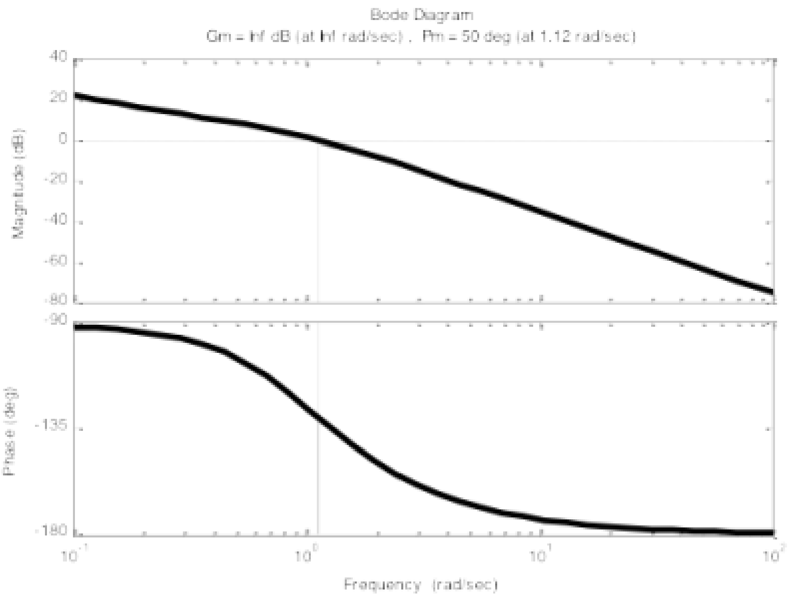

Fig. 8 Bode plot for open-loop system with final PI controller

Settling time was measured from unity step response (Fig. 9) and the value is T_{s_fin}=6.15s what is very close to desired settling time T_s=6s.

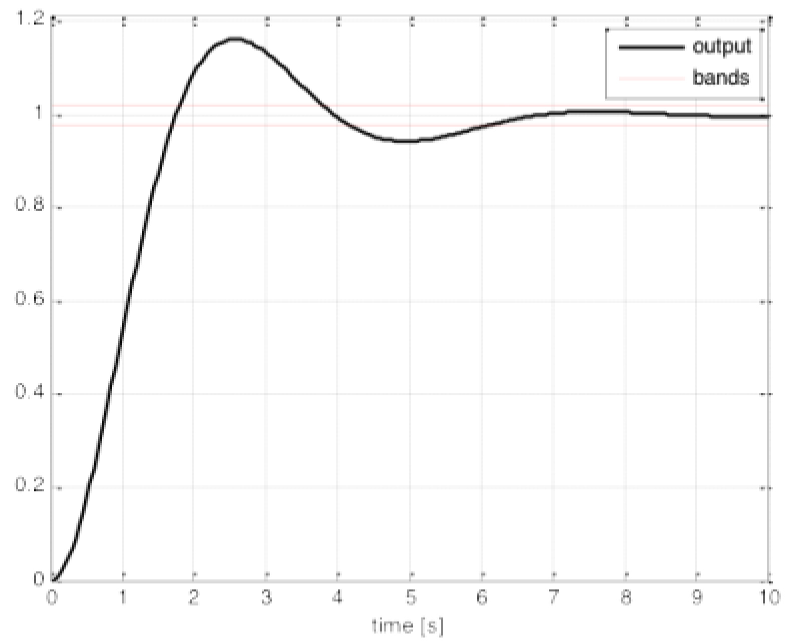

Fig. 9 Step response for closed loop system with final PI controller

5. Conclusions

In this paper modification of Neimark D-partition method was presented. This controller design approach ensures not only stability but performance in term of phase margin and desired settling time too. The developed frequency domain design technique is graphical, interactive and insightful and it is useful for stable systems. Theoretical results have been verified on case study.

Acknowledgments

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